

Enrollment No: \_\_\_\_\_

Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Summer Examination-2016

Subject Name: Mathematics-II

Subject Code: 4SC02MTC1

Branch: B.Sc.(All)

Semester: 2    Date: 13/05/2016

Time: 10:30 To 1:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1**                      **Attempt the following questions:** **(14)**
- a) State De'Moivre's theorem. **(02)**
  - b) Prove that  $\cosh^2 x - \sinh^2 x = 1$ . **(02)**
  - c) Evaluate:  $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$ . **(02)**
  - d) Solve:  $(D^2 + 6D + 9)y = 0$ . **(02)**
  - e)  $\frac{1}{f(D)}(e^{mx} x^2) = e^{mx} \frac{1}{f(D+m)} x^2$ . Determine whether the statement is true or false? **(01)**
  - f) Define: Oscillatory Sequence. **(01)**
  - g) Find polar form of  $1 - i\sqrt{3}$ . **(01)**
  - h) Write a condition that the general equation of second degree represents a cone. **(01)**
  - i) Any line on the surface of a cylinder is called its \_\_\_\_\_. **(01)**
  - j)  $\frac{1}{D-m} X = e^{-mx} \int X e^{mx} \, dx$ . Determine whether the statement is true or false? **(01)**

Attempt any four questions from Q-2 to Q-8

- Q-2**                      **Attempt all questions** **(14)**
- a) Solve:  $x^6 - 1 = 0$ . **(05)**
  - b) Expand  $\frac{\sin 6\theta}{\sin \theta}$  in terms of increasing power of  $\cos \theta$ . **(05)**
  - c) Prove that  $\tan^{-1} \left( i \log \frac{a-ib}{a+ib} \right) = \frac{2ab}{a^2-b^2}$ . **(04)**
- Q-3**                      **Attempt all questions** **(14)**
- a) Find equation of cylinder whose generators intersect the conic  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0; z = 0$ . **(05)**
  - b) Prove that  $\cos^8 \theta = \frac{1}{128} [\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$ . **(05)**
  - c) Prove that  $\sinh^{-1}(\tan x) = \log \tan \left( \frac{\pi}{4} + \frac{x}{2} \right)$ . **(04)**



- Q-4**      **Attempt all questions**      (14)
- a) Find equation of cone whose vertex is the point (1, 1, 1) and whose guiding curve is  $x^2 + z^2 = 4, y = 0$ .      (05)
- b) Prove that  $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$ .      (05)
- c) Find the equation of the enveloping cylinder of the sphere  $x^2 + y^2 + z^2 = 25$ , whose generators are parallel to the line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ .      (04)
- Q-5**      **Attempt all questions**      (14)
- a) Check whether the equation  $x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6xz + 8x - 19y - 2z - 20 = 0$  represent a cone with vertex (1, -2, 3).      (05)
- b) Prove that equation of right circular cylinder whose axis is the line  $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$  and radius  $r$  is  $(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 - \frac{[l(x-\alpha)+m(y-\beta)+n(z-\gamma)]^2}{l^2+m^2+n^2} = r^2$       (05)
- c) Prove that  $\lim_{n \rightarrow \infty} \frac{1+\frac{1}{3}+\frac{1}{5}+\dots+\frac{1}{2n-1}}{n} = 0$ .      (04)
- Q-6**      **Attempt all questions**      (14)
- a) Prove that  $\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$ .      (05)
- b) Solve:  $y'' - 5y' + 6y = 0; y(1) = e^2, y'(1) = 3e^2$ .      (05)
- c) Evaluate: i)  $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^8 x \, dx$ , ii)  $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^5 x \, dx$ .      (04)
- Q-7**      **Attempt all questions**      (14)
- a) Solve:  $3 \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - y = e^{\frac{x}{2}} + 2e^{3x}$ .      (05)
- b) Solve:  $(D^3 + 3D^2 + 2D)y = x^2$ .      (05)
- c) Solve:  $(D^3 + 1)y = 0$ .      (04)
- Q-8**      **Attempt all questions**      (14)
- a) Solve:  $(x^2 D^2 - 3xD + 4)y = x^2$ .      (05)
- b) Solve:  $\frac{dx}{dt} = 5x + y, \frac{dy}{dt} = y - 4x$ .      (05)
- c) Identify the given surface  $9x^2 + 4y^2 - 9z^2 - 18x - 8y - 18z = 32$ .      (04)

